$B \to K(K^*)X$: DECAY DISTRIBUTIONS, BRANCHING RATIOS AND CP ASYMMETRIES

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We discuss an improved theoretical description of the semi-inclusive B meson decays $B \to K(K^*)X$. The decay distributions are calculated. Their branching ratios are found to be appreciable. The CP asymmetries in the neutral B modes $\bar{B}^0 \to K^-(K^{*-})X$ are sizable. An observation of direct CP violation and a measurement of γ may come from these neutral B modes.

1 Introduction

The semi-inclusive charmless hadronic decays $B \to K(K^*)X$, where X is a hadronic recoil system containing no charmed and strange particles, are useful for studying direct CP violation and determining the weak phase γ [1]. Direct CP violation can also be studied with exclusive or inclusive charmless hadronic B decays. Although theoretical uncertainties in inclusive decays may be small, it is experimentally hard to identify final states inclusively, while exclusive decays have a clear experimental signature, but the theoretical calculation is not as clean. Semi-inclusive hadronic decays, which lie somewhere between inclusive and exclusive hadronic decays, have a small theoretical uncertainty and a clear experimental signature, thereby providing an interesting channel for studying direct CP violation. Here[a] we report on a recent study[2] of $B \to K(K^*)X$ decays.

Charmless hadronic $B \to K(K^*)X$ decays involve two types of amplitudes: $b \to u$ tree amplitudes and $b \to s$ penguin amplitudes. Direct CP violation arises due to the interference between two or more participating amplitudes with different weak and strong phases for a single decay mode. The effective Hamiltonian for charmless hadronic B decays with $\Delta S = 1$ is given by

$$H_{eff} = \frac{G_F}{\sqrt{2}}\{V_{ub}V_{us}^*(c_1 O_1 + c_2 O_2 + \sum_{n=3}^{11} c_n O_n) + V_{cb}V_{cs}^* \sum_{n=3}^{11} c_n O_n\},$$

(1)

where O_n are local four-quark and magnetic moment operators and c_n the corresponding Wilson coefficients. Both c_n and the matrix elements of O_n depend on the renormalization scheme and scale. The effective theory based on the Wilson operator product expansion provides a framework to separate the short- and long-distance strong interaction. The short-distance strong interaction effects above the scale $\mu \sim m_b$ are incorporated in the Wilson coefficients c_n. The long-distance strong interaction effects below the scale μ are encoded

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in the hadronic matrix elements of the local operators $< XK(K^*)|O_n|B >$.

The short-distance coefficients c_n are well known. They have been calculated up to the next-to-leading-order corrections [3]. Factorization has often been employed to calculate hadronic matrix elements. Recent theoretical work [4,5,6] has justified factorization in the heavy quark limit in the case that the ejected particle from the B decay is a light meson or an onium. Perturbative QCD corrections to factorization can be computed in the heavy quark limit. The separation of short and long distance QCD using the effective Hamiltonian together with QCD factorization, light cone expansion and heavy quark effective theory leads to an improved theoretical description of $B \to K(K^*)X$ decays.

2 Initial Bound State Effects

We choose to study $\bar{B}^0 \to K^-(K^{*-})X$ and $B^- \to \bar{K}^0(\bar{K}^{*0})X$. The factorized matrix elements for these processes do not involve the transition form factors, eliminating a potential theoretical uncertainty. Using the effective Hamiltonian (1) and QCD factorization [4], the differential decay rate for $B \to KX$ in the B rest frame is given by

$$d\Gamma = \frac{1}{2m_B}\frac{d^3\mathbf{P}_K}{(2\pi)^3 2E_K}(-2f_K^2)$$
$$\times \{|\alpha|^2 P_K^\mu P_K^\nu(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta}) + |\beta|^2 g_{\alpha\beta}\}$$
$$\times \int d^4y\, e^{iy\cdot P_K}[\partial^\alpha \Delta_{q'}(y)] < B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B >,$$

(2)

where $q' = u$ and d for $\bar{B}^0$ and B^-, respectively. After factorization the long-distance QCD effects on $B \to KX$ decays are contained in two matrix elements: $< K|\bar{s}\gamma^\mu(1 - \gamma_5)q'|0 > = if_K P_K^\mu$ defining the decay constant f_K for a K meson and $< B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B >$ incorporating the effect of the initial b quark bound state in the B meson. The K meson decay constant is known from experiment [7]: $f_K = 159.8 \pm 1.5$ MeV. We need to know another hadronic matrix element $< B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B >$.

We use the light cone expansion method to handle the initial bound state effect. Since the dominant contribution to the decay rate comes from the light cone region, the light cone expansion can be used to systematically calculate the matrix element. In the leading twist approximation, $< B|\bar{b}(0)\gamma^{\beta}U(0,y)b(y)|B > = 2P_B^{\beta}\int d\xi\, e^{-i\xi y\cdot P_B}f(\xi)$, where $f(\xi)$ is the b-quark distribution function [8,9]. We then obtain the decay distribution as a function of E_K

$$\frac{d\Gamma(B \to KX)}{dE_K} = \frac{f_K^2 E_K}{2\pi m_B}\left(4|\alpha|^2 E_K^2 + |\beta|^2\right)f(\frac{2E_K}{m_B}). \quad (3)$$

Carrying out similar calculations, we obtain the differential decay rate for the $B \to K^*X$ decay

$$\frac{d\Gamma(B \to K^*X)}{dE_{K^*}} = \frac{f_{K^*}^2 E_{K^*}^3}{2\pi m_B}4|\alpha_*|^2 f(\frac{2E_{K^*}}{m_B}). \quad (4)$$

The distribution of $E_{K^{(*)}}$ is a delta function with the peak at $E_{K^{(*)}} = m_b/2$ in the free quark decay $b \to K(K^*)q'$ in the b rest frame. Gluon bremsstrahlung and initial bound state effect smear the spectrum. The leading effect of the initial b-quark bound state is described by the distribution function $f(\xi)$. It is important to notice that the same distribution function also encodes the leading nonperturbative QCD effects in $B \to X\gamma$ [9] and inclusive semileptonic decays $B \to Xl\bar{\nu}$ [8]. Universality of the distribution function enhances predictive power: The distribution function can be measured in one process and then used to make predictions for all other processes.

Several important properties of the distribution function are already known in QCD [8,9]. When integrating ξ from 0 to 1, $\int_0^1 d\xi f(\xi)$ must give 1 due to current conservation. If the decay can be considered to be a free b quark decay, then the b quark field is given by $b(y) = e^{-iy\cdot P_b}b(0)$, one obtains

$$f(\xi) = \delta(\xi - \frac{m_b}{m_B}). \quad (5)$$

We can also estimate the mean $< \xi > = \int_0^1 d\xi \xi f(\xi)$ and the variance $\sigma^2 = \int_0^1 d\xi \xi^2 f(\xi) - < \xi >^2$ using heavy quark effective theory [10]. They are given by [9,8]

$$< \xi > = \frac{m_b}{m_B}[1 + \frac{5}{6m_b^2}(\mu_\pi^2 - \mu_g^2)],$$

$$\sigma^2 = \frac{\mu_\pi^2}{3m_B^2}, \quad (6)$$

where

$$\mu_g^2 = \frac{1}{4m_B} < B|\bar{h}g_s G_{\mu\nu}\sigma^{\mu\nu}h|B >,$$

$$\mu_\pi^2 = -\frac{1}{2m_B} < B|\bar{h}(iD_T)^2 h|B >, \quad (7)$$

are two parameters of heavy quark effective theory. The small value for σ^2 implies that the distribution function is sharply peaked around m_b/m_B.

The detailed form of the distribution function is not yet known. We use the following general parametrization for the distribution function for our numerical analysis:

$$f(\xi) = N\frac{\xi(1-\xi)^c}{[(\xi-a)^2 + b^2]^d}, \quad (8)$$

where N is a normalization constant which guarantees $\int_0^1 d\xi f(\xi) = 1$. The four parameters (a, b, c, d) respect all the known properties of the distribution function.

We have also calculated the initial bound state effect by directly using heavy quark effective theory. The $1/m_b$ expansion relates the matrix element to the parameters of heavy quark effective theory [11]. In terms of the parameters of heavy quark effective theory, the decay rates are given by

$$\Gamma(B \to KX) = \frac{f_K^2}{8\pi}m_b[|\alpha|^2 m_b^2(1 + \frac{7}{6}\frac{\mu_g^2}{m_b^2} - \frac{53}{6}\frac{\mu_\pi^2}{m_b^2})$$

$$+|\beta|^2(1 - \frac{\mu_\pi^2}{2m_b^2} + \frac{\mu_g^2}{2m_b^2})], \quad (9)$$

$$\Gamma(B \to K^*X) = \frac{f_{K^*}^2}{8\pi}m_b|\alpha_*|^2 m_b^2$$

$$\times(1 + \frac{7}{6}\frac{\mu_g^2}{m_b^2} - \frac{53}{6}\frac{\mu_\pi^2}{m_b^2}). \quad (10)$$

The free quark decay $b \to K(K^*)q'$ results are reproduced in the heavy quark limit.

3 Results and Discussions

We have calculated the decay distributions, CP-averaged branching ratios, and CP asymmetries in $B \to K(K^*)X$ decays. The phenomenological treatment has been improved with better theoretical understanding. There are still several sources of theoretical uncertainties, including the input parameters, the meson light cone distribution amplitudes, the distribution function, the renormalization scale, and the unknown power corrections. All the results obtained should be understood as valid within the theoretical uncertainties.

The distribution of the $K(K^*)$ energy calculated in the heavy quark effective theory approach is the same delta function with the peak at $E_{K^{(*)}} = m_b/2$ as in the free quark decay $b \to K(K^*)q'$. The light cone expansion approach is capable of accounting for the initial bound state effect on the decay distribution. The resulting spectrum spreads over the full kinematic range $0 \le E_{K^{(*)}} \le m_B/2$ and depends strongly on the form of the distribution function. However, if the cut $E_{K^{(*)}} > 2.1$ GeV is applied to suppress the background, more than
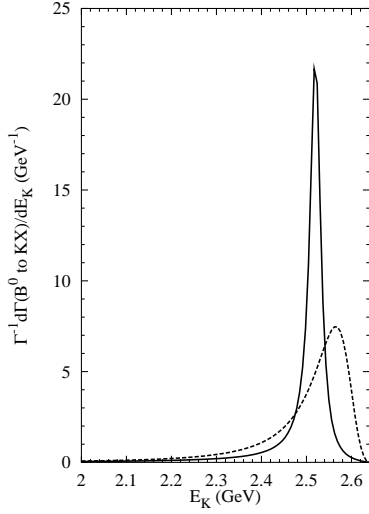
Figure 1: Kaon energy spectrum in $\bar{B}^0 \to K^- X$. The solid and dashed curves correspond to two very different forms of the distribution function $f(\xi)$, respectively.

97% of events will survive. For illustration, the kaon energy spectrum in $\bar{B}^0 \to K^- X$ is shown in Fig. 1, computed in the light cone expansion approach assuming $\gamma = 60°$.

One can also calculate the hadronic invariant mass spectrum $d\Gamma/dM_X$ in $B \to KX$. M_X is related with E_K in the B rest frame through $M_X^2 = (P_B - P_K)^2 = m_B^2 - 2m_B E_K + m_K^2$. The M_X distribution can be calculated by using $d\Gamma/dM_X = (M_X/m_B)d\Gamma/dE_K$ and the above kinematic relation; $d\Gamma/dE_K$ is given in Eq. (3). Similarly, one can also calculate the M_X distribution in $B \to K^* X$.

We show the CP-averaged branching ratios, in Figs. 2-5, and the CP asymmetries, in Figs. 6-9, in $B \to K(K^*)X$ as a function of the CP violating phase γ. The solid curves are the results from the light cone expansion, while the dashed curves are from the free b quark decay approximation. The initial bound state effects encoded in the distribution function cancel in the CP asymmetries in $B \to K^* X$, so that the solid and dashed curves coincide in Figs. 8 and 9.

We find that the initial bound state effects on the branching ratios and CP asymmetries are small. In the light cone expansion approach, the CP-averaged branching ratios are increased by about 2% with respect to the free b-quark decay. For $\bar{B}^0 \to K^-(K^{*-})X$, the CP-averaged branching ratios are sensitive to the phase γ and the CP asymmetry can be as large as 7% (14%), whereas for $B^- \to \bar{K}^0(\bar{K}^{*0})X$ the CP-averaged branching ratios are not sensitive to γ and the CP asymmetries are small ($< 1\%$). The CP-averaged branching ratios are predicted to be in the ranges $(0.53 \sim 1.5) \times 10^{-4}$ $[(0.25 \sim 2.0) \times 10^{-4}]$ for $\bar{B}^0 \to K^-(K^{*-})X$ and $(0.77 \sim$

$0.84) \times 10^{-4}$ $[(0.67 \sim 0.74) \times 10^{-4}]$ for $B^- \to \bar{K}^0(\bar{K}^{*0})X$, depending on the value of the CP violating phase γ. In the heavy quark effective theory approach, the branching ratios are decreased by about 10% and the CP asymmetries are not affected in comparison with the free b-quark decay. The three estimates (free quark decay approximation, light cone expansion and heavy quark effective theory method) all give the same order of magnitudes for the branching ratios and CP asymmetries. The branching ratios for $B \to K(K^*)X$ are of order 10^{-4} and the CP asymmetries in the neutral B modes $\bar{B}^0 \to K^-(K^{*-})X$ are sizable and can be measured at the B factories.

Acknowledgments

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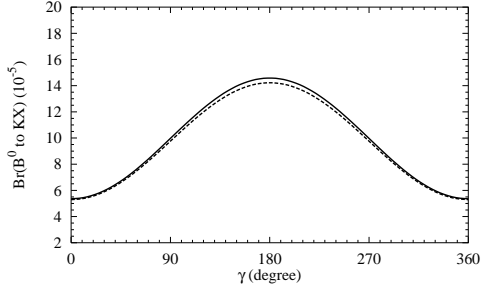11. J.P. Ma, Phys. Lett. **B 488**, 55 (2000).

Figure 2: CP-averaged branching ratio for $\bar{B}^0 \to K^- X$.

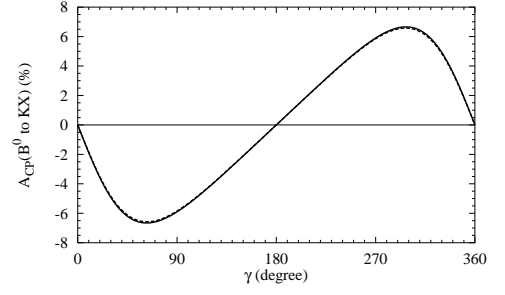Figure 3: CP-averaged branching ratio for $B^- \to \bar{K}^0 X$.

Figure 4: CP-averaged branching ratio for $\bar{B}^0 \to K^{*-} X$.
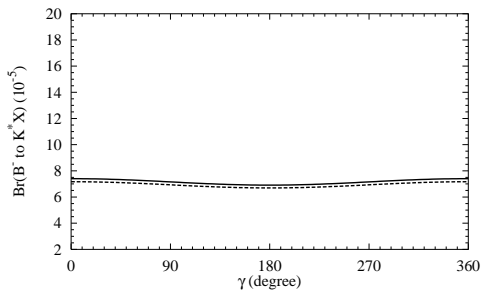

Figure 5: CP-averaged branching ratio for $B^- \to \bar{K}^{*0} X$.

Figure 6: CP asymmetry in $\bar{B}^0 \to K^- X$.
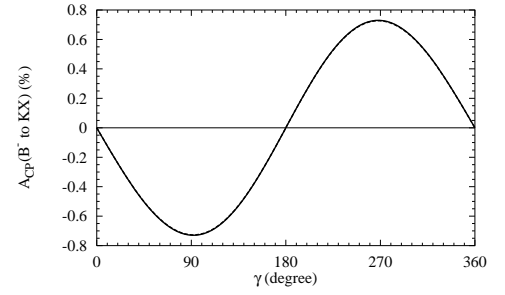
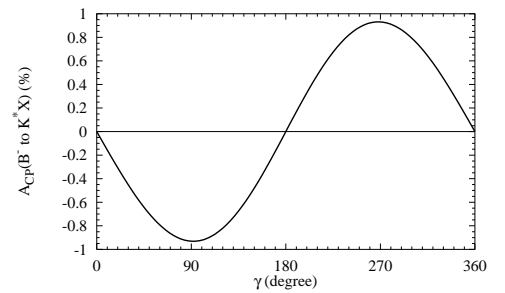Figure 7: CP asymmetry in $B^- \to \bar{K}^0 X$.
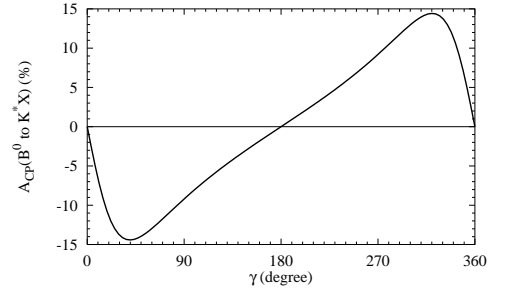
Figure 8: CP asymmetry in $\bar{B}^0 \to K^{*-} X$.

Figure 9: CP asymmetry in $B^- \to \bar{K}^{*0} X$.